part of the recycled material entering the jet; Q_p'', part of the recycled material entering the bulk; Q_c, suspension flow rate; ρ_j, ρ, distribution densities in the jet and bulk, respectively; r, particle radius; S, particle surface; V, particle volume; t, time; α, fraction of recycled material supplied to the jet; n_{un}, product unloading rate; n_p, recycled-material supply rate; τ_i, instant i of the instantaneous recycled material loading; ν_j, instant j of instantaneous product unloading; Q_p, Q_{un}, flow rates of the recycled material and the final product; N_0, initial number of particles in the apparatus; ΔQ_p, ΔQ_{un}, volumes of the recycled material and product supplied to and withdrawn from the apparatus at a fixed instant.

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IMPROVEMENT OF THE LIMITING RELATIVE FRICTION LAW ON A PERMEABLE PLATE WITH BLOWN GAS

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The authors have improved the correlation for the effect of finite Reynolds number on the critical parameters of blowing and the limiting relative friction law in the incompressible turbulent boundary layer with blowing of gas at the wall.

Asymptotic turbulent boundary-layer theory was used in [1] to obtain the following formulas for the limiting relative friction law and the critical blowing parameters:

for $\psi < 1$

$$\Psi_\infty = \frac{4}{b_1(1-\psi)}\left[\ln\frac{\sqrt{(1-\psi)(1+b_1)}+\sqrt{b_1}}{\sqrt{1-\psi}+\sqrt{b_1\psi}}\right]^2, \tag{1}$$

$$b_{cr\infty} = \frac{1}{1-\psi_1}\left(\ln\frac{1+\sqrt{1-\psi_1}}{1-\sqrt{1-\psi_1}}\right)^2; \tag{2}$$

for $\psi > 1$

$$\Psi_\infty = \frac{4}{b_1(\psi-1)}\left[\operatorname{arctg}\sqrt{\frac{b_1}{(\psi-1)(b_1+1)}} - \operatorname{arctg}\sqrt{\frac{b_1\psi}{\psi-1}}\right]^2, \tag{3}$$

$$b_{cr\infty} = \frac{1}{\psi_1-1}\left(\arccos\frac{2-\psi_1}{\psi_1}\right)^2; \tag{4}$$

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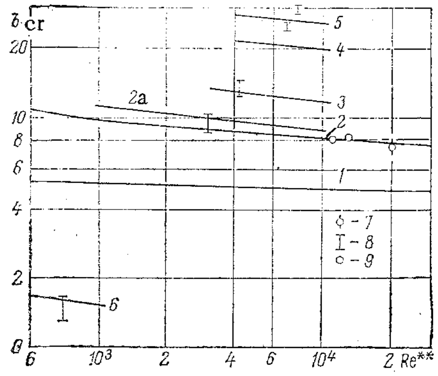

Fig. 1

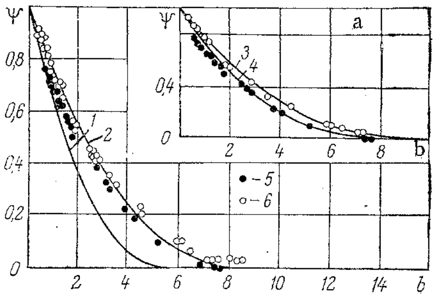

Fig. 2

Fig. 1. Comparison of measured and calculated values of the critical displacement parameters. Theory: 1) calculated from Eq. (10); 2-6) calculated from Eq. (16) using Eqs. (2), (4) and (6); 2) blowing of nitrogen into air ($\psi_1 = 1$); 3) blowing of carbon dioxide ($\psi_1 = 0.66$); 4) blowing of krypton ($\psi_1 = 0.34$); 5) blowing of Freon-12 ($\psi_1 = 0.24$); 6) blowing of helium ($\psi_1 = 7.3$). Experiment: 7) [2]; 8) [3]; 9) [4]; 2a) calculated from Eq. (14) for $\psi_1 = 1$.

Fig. 2. Variation of the relative friction law at constant Re** as a function of the blowing parameter for $\psi_1 = 1$: 1) calculated from Eq. (11), allowing for Eqs. (6) and (10) for $Re^{**} = 5\cdot10^3$; 2) calculated from Eq. (17); a) 3, 4) calculated from Eq. (11), allowing for Eqs. (6) and (15) for $Re^{**} = 2\cdot10^3$ and $Re^{**} = 10^4$, respectively; experiment: 5) [2]; 6) [6].

for $\psi = 1$

$$\Psi_\infty = (1 - b/4)^2 = (1 - b/b_{cr\,\infty})^2, \tag{5}$$

$$b_{cr\,\infty} = 4. \tag{6}$$

For the region of finite Reynolds number Re** the critical blowing parameter is determined by the formula [1]

$$\sqrt{b_{cr}} = \frac{1}{z}\int_0^1 \sqrt{\frac{\rho}{\rho_0}\,\frac{1+2\xi}{\omega}}\,d\omega. \tag{7}$$

Reference [1] assumed z = 1 as a first approximation, and for the function $\xi(\omega)$ the formula

$$\sqrt{\omega} = (1 + 1{,}25\sqrt{0{,}5c_{f0}}\ln\xi), \tag{8}$$

$$\sqrt{2/c_{f0}} = 2.5\ln Re^{**-0{,}14} + 3{,}8. \tag{9}$$

The results calculated from Eq. (7), allowing for Eqs. (8) and (9) and with z = 1, were approximated in [1] by the expression

$$b_{cr} = b_{cr\,\infty}(1 + 0.83\,Re^{**-0{,}14}), \tag{10}$$

and it was proposed to determine the relative friction law, in analogy with Eq. (5), from the formula

$$\Psi = (1 - b/b_{cr})^2, \tag{11}$$

in which b_{cr} is found from Eq. (7). The literature presently has reliable data which can be used to verify Eqs. (7) and (11).

The value of b_{cr} was determined in [2] for the first time by means of a floating sensor, when the friction force at the wall is zero, in [3] by an interferometric method, in [4] by displacement of the thermal boundary layer when the temperature of the blown gas is below that of the flow gas, and in [5] by a chemical visualization method. However, it should be noted that the test data of [5] cannot be identified with critical values of the displacement parameter under homogeneous blowing of gas. Those data were obtained with blowing of a weak aqueous solution of alkali into a concentrated aqueous solution of salt or citric acid, when the density ratio was $\rho_0/\rho_+ = 1.2$-1.3 and Pr = 6. Figure 1 compares Eq. (10) with the test data. It can be seen that Eq. (10) does not adequately account for the influence of the finite value of Re** on the critical displacement parameters. And this is why, as can be seen in Fig. 2, the test data on friction coefficients also diverge from the Eq. (11) theory for $b > 1.5$. Figure 2 uses the most reliable and accurate measurements of friction coefficients, obtained in [2, 6] in isothermal subsonic flow of gas over a flat plate.

In these references special measures were taken to minimize possible errors associated with nonuniformity of the plate permeability (2% in [2] and 6% in [6]), with the presence of a pressure gradient (±0.5 mm of water column), the influence of wall roughness, and of increased degree of flow turbulence (0.2%). The orifice plates for measuring the flow rate of blown gas were calibrated to an accuracy of ±1%. The velocity profiles were measured with total head tubes which included special structural features to reduce the error in measuring velocities near the wall. A direct method was used in [2] for measuring friction at the permeable wall using a floating element. The floating elements used earlier in [7] had appreciable defects: rather large gaps between the sensor and the permeable plate, varying from 0.19 to 1.9 mm as the blowing increased [8]. Therefore, the sensors in [7] had considerable errors in measuring small friction forces at large blowing levels, and did not allow measurement of zero friction force under conditions close to displacement of the boundary layer.

The floating element was improved in [2]. The gap around the element, of dimensions 50.8×50.8 mm^2 was reduced to 0.076 mm, and was held fixed with increase of blowing intensity by an automatic centering system. With this sensor the authors were able, for the first time, to record friction forces close to zero at levels of blowing parameter b = 7.3-7.8. The friction coefficients with gas blowing were calculated in [6] by an integral method and by the viscous sublayer method from the measured velocity profiles. The experimental facility allowed tests to be conducted at constant and variable gas blowing levels along a flat plate, maintaining the cases $\bar{j}_w = \text{const}$, $b = \text{const}$, $\bar{j}_w = kx$, $\bar{j}_w = k_1\sqrt{x}$.

The error in determining the friction coefficients in [6] is associated only with inaccuracy in measuring the velocity profiles and with the method used to calculate the friction forces. One can agree with the authors of [2, 6] that they obtained reliable data on friction coefficients on a permeable plate with blowing of a homogeneous gas into an isothermal boundary layer. Because of the high accuracy in measuring the friction coefficients in [2, 6] one can note the stratification of the test points according to Re**. Figure 2a shows the test points of [2, 6] on relative friction coefficients for two values of Reynolds number: Re** = $(2$-$3)\cdot 10^3$ and Re** = $(1$-$2)\cdot 10^4$. The stratification in terms of Re** is clear, although it is small in absolute magnitude, less than 12% for a change of Re** of almost an order of magnitude.

The literature already contains a series of experimental papers [7, 9, 10] in which the measured friction coefficients are close to those obtained in [2, 6]. But, in the opinion of the authors of [8], the work conducted earlier on the same facility as [2] is unreliable for a number of reasons. To all appearances one should also judge as unreliable the measurements of [11] which differ from those of [2, 6]. The author of [11] himself stated low accuracy in measuring friction forces at the wall using a Preston tube at large blowing levels. An interesting paper is [12], where the test data refer to flow over a permeable rough plate with small negative pressure gradients.

Taking account of what has been said above, one can conclude that there is no conflict in regard to measured friction coefficients on a permeable plate washed by a subsonic isothermal zero-gradient gas boundary layer, and that the test data of [2, 6] should be taken as standard.

Improvement of the Critical Blowing Parameters. In Eq. (7) we shall take z to be z_0 at the surface under the actual conditions of flow over the plate in [1]:

$$z = z_0 = 1 - \omega_{10} = 1 - \varphi_{10}\sqrt{\Psi_\rho c_{f0}/2}. \tag{12}$$

It follows from the test data of [13-15] that $\varphi_{10} = 7, 8$ and for Ψ_ρ we use the formula of [1]:

$$\sqrt{\Psi_0} = 2(1+\sqrt{\psi_1})^{-1}. \tag{13}$$

The theoretical values of b_{cr} can be approximated in the range $\mathrm{Re}^{**} = 10^3 - 10^5$ and $\psi_1 = 0.1 - 8$, with an error of less than 10%, by the formula

$$b_{cr} = b_{cr\infty}(1 + 6.5\psi_1^{-0.35}\mathrm{Re}^{**-0.18}). \tag{14}$$

Figure 1 shows values calculated from Eq. (14) for $\psi_1 = 1$ (curve 2a). It can be seen that this calculation gives somewhat high values of b_{cr} compared with experiment. By introducing a correction factor into Eq. (14) one can get satisfactory agreement between test data and theory ($\psi_1 = 1$) using the formula (curve 2)

$$b_{cr} = b_{cr\infty}(1 + 5.3\,\mathrm{Re}^{**-0.18}). \tag{15}$$

Then the results of calculating b_{cr} from Eq. (1) in the range $\mathrm{Re}^{**} = 10^3 - 10^5$, $\psi_1 = 0.1 - 8$, and allowing for the correction made, can be approximated, to within 10%, by the expression

$$b_{cr} = 4\psi_1^{-0.6}(1 + 5.3\psi_1^{-0.35}\mathrm{Re}^{**-0.18}), \tag{16}$$

in which b_{cr} is determined from Eqs. (2), (4), and (6).

The literature has no test data on critical blowing parameters under appreciably nonisothermal conditions. However, there are experiments [3] on b_{cr} with blowing of foreign gases under isothermal conditions. As was shown in [1], the parameter ψ_1 for these conditions is equal to the ratio of the gas constants of the blown gas and the primary flow: $\psi_1 = R_0/R_+$. Figure 1 compares values calculated from Eq. (16) with test data for various values of ψ_1. One can see satisfactory agreement between theory and experiment.

As can be seen from Fig. 2a, there is also good agreement between the measured relative friction coefficients and values calculated from Eqs. (11) and (15). For practical calculations in the range $\mathrm{Re}^{**} = (0.08\text{-}2)\cdot 10^4$ the quantity b_{cr} for $\psi_1 = 1$ can be assumed to be constant and $b_{cr} = 8$. The expression

$$\Psi = (1 - b/8)^2 \tag{17}$$

describes the test data (Fig. 2) quite well. It should be noted that the test data on the relative friction coefficients at variable blowing levels used in [6] and shown in Fig. 2 do not differ appreciably in the proposed formulation from the data for the case $\bar{j}_w = \text{const}$. This speaks to the conservative nature of the relative friction law at constant Re^{**} as a function of changing boundary conditions.

In the literature one often finds a comparison of the relative friction law as a function of the blowing parameter at constant Re_x, and not Re^{**}. Therefore, in Fig. 3 we have shown the test data of [2, 6] evaluated in the form of the dependence of Ψ_x on b_x, in which the quantity c_{f0x} was calculated from the formula $c_{f0x} = 0.0592\mathrm{Re}_x^{-0.2}$ [6], and not from Eq. (9). Figure 3 shows a calculation from the expression

$$\Psi_x = (1 - b_x/b_{x\,cr})^2 = (1 - b_x/5.3)^2, \tag{18}$$

which uses the connection between b_{cr} and b_{crx} in the form $b_{crx} = b_{cr}^{0.8}$ obtained in [1]. It can be seen from Fig. 3 that Eq. (18) agrees satisfactorily with experiment.

Improvement of the Relative Friction Law in Nonisothermal Conditions. In blowing of a gas with initial temperature T' differing from the temperature of the primary flow T_0, there is a single-valued connection between ψ and b_{T_1}, obtained from the energy balance at the wall for $q_l = 0$ [1]:

$$\psi = \frac{1 + b_{T1}\psi_1}{1 + b_{T1}}. \tag{19}$$

It follows from Eq. (19) that in the absence of blowing of gas ($b_{T_1} = 0$) we have $\psi = 1$ and correspondingly, $\Psi_\infty = 1$. This should be borne in mind when Eqs. (1) and (3) are used for practical calculations.

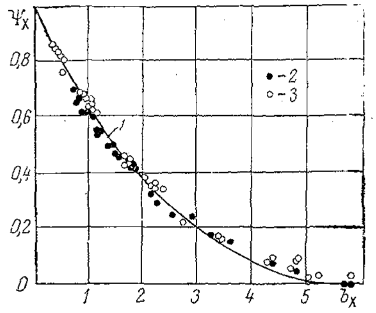

Fig. 3

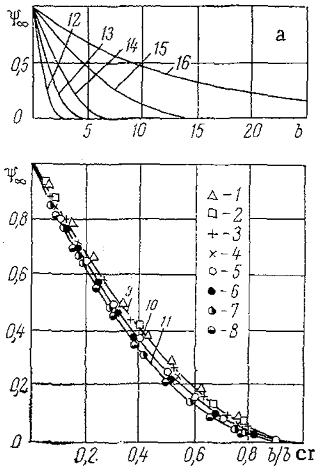

Fig. 4

Fig. 3. Variation of the relative friction law for constant Re_x as a function of the blowing parameter b_x: 1) calculation from Eq. (18); experiment: 2) [2]; 3) [6].

Fig. 4. Variation of the relative friction law as a function of the relative blowing parameter: 1-8), 12-16) calculation from Eqs. (1), (3) and (5) for various values of ψ_1: 1) $\psi_1 = 0.1$; 2) 0.2; 3) 0.5; 4) 0.9; 5) 1.0; 6) 1.5; 7) 1.8; 8) 2; 9-11) calculation from Eqs. (20)-(22); a) variation of Ψ_∞ as a function of the blowing parameter b: 12) $\psi_1 = 2$; 13) 1.0; 14) 0.5; 15) 0.1; 16) 0.

By substituting Eq. (19) into Eqs. (1) and (3) one can obtain the relative friction law as a function of b for various ratios ψ_1 known from the original data. The results of the calculations can be approximated, to an accuracy within 5%, by the formulas: for $\psi_1 = 0.2$-0.9.

$$\Psi_\infty = (1 - b/b_{cr\,\infty})^{1.8}; \qquad (20)$$

for $\psi_1 = 0.9$-1.5

$$\Psi_\infty = (1 - b/b_{cr\,\infty})^{2}; \qquad (21)$$

for $\psi_1 = 1.5$-2

$$\Psi_\infty = (1 - b/b_{cr\,\infty})^{2.2}. \qquad (22)$$

Figure 4 shows a comparison of calculations using Eqs. (1), (3), and (5) and Eqs. (20)-(22). As can be seen from Fig. 4a the relative friction law is stratified appreciably in terms of the parameter ψ_1 in the coordinates $\Psi = f(b)$. The literature has no test data on friction in substantially nonisothermal conditions of flow over permeable surfaces with blowing of gas.

NOTATION

$\Psi = c_f/c_{f0}$, $\Psi_x = c_f/c_{f0x}$, relative friction laws for constant values of Re** and Re_x, respectively; c_f, c_{f0}, c_{f0x}, friction coefficients; $b = 2\bar{j}_w/c_{f0}$, $b_1 = 2\bar{j}_w/c_f$, $b_x = 2\bar{j}_w/c_{f0x}$, $b_{T_1} = \bar{j}_w/St$, blowing parameters; $\bar{j}_w - j_w/\rho_0 u_0$, relative flow rate of blown gas; $\psi = i_w/i_0$, ratio of the gas enthalpies at the temperatures of the wall and the primary flow; $Re^{**} = \rho_0 u_0 \delta^{**}/\mu_w$, $Re_x = \rho_0 u_0 x/\mu_0$, Reynolds numbers; δ^{**}, momentum loss thickness, m; $\omega = u/u_0$, $\omega_{10} = u_1/u_0$, $\varphi_{10} = u_1/u_*$, $u_* = u_0\sqrt{c_f/2}$, relative velocities; u_1, velocity at the edge of the laminar sublayer; $\xi = y/\delta$, relative transverse coordinate; δ, boundary-layer thickness, m; cr, critical; x, longitudinal coordinate, m.

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